

Series of 0.0-ε

→ حل المعادلة التفاضلية باستخدام المتسلسلة

$$1 \cdot \ddot{y} + P(x) \dot{y} + Q(x) y = F(x)$$

1 0-P $P(0) \neq \infty$, $Q(0) \neq \infty$

$$y = \sum_{n=0}^{\infty} a_n x^n$$

ex $\ddot{y} + y = 0$, $x_0 = 0$

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~~ex~~ $P(x) = 0$, $Q(x) = 1$

$P(0) = 0 \neq \infty$, $Q(0) = 1 \neq \infty \Rightarrow (0-P)$

$$y = \sum a_n x^n$$

$$\dot{y} = \sum n a_n x^{n-1} \Rightarrow \ddot{y} = \sum n(n-1) a_n x^{n-2}$$

$$\sum n(n-1) a_n x^{n-2} + \sum a_n x^n = 0$$

→ coeff. of x^0

$$2a_2 + a_0 = 0 \Rightarrow a_2 = \frac{-a_0}{2}$$

→ coeff. of x^1

$$6a_3 + a_1 = 0 \Rightarrow a_3 = \frac{-a_1}{6}$$

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→ coeff. of x^n :-

$$(n+2)(n+1)a_{n+2} + a_n = 0$$

$$a_{n+2} = \frac{-a_n}{(n+2)(n+1)}$$

$$\underline{n=2} \quad a_4 = \frac{-a_2}{12} = \frac{a_0}{24}$$

$$\underline{n=3} \quad a_5 = \frac{-a_3}{20} = \frac{a_1}{120}$$

$$y = a_0 + a_1 x + \frac{-a_0}{2} x^2 + \frac{-a_1}{6} x^3 + \frac{a_0}{24} x^4 \dots \dots \dots$$

$$t = x - x_0 \leftarrow x_0 \neq 0 \quad \textcircled{1} \quad \text{الأفكار :}$$

$$\boxed{\text{ex}} \quad y'' + xy' + y = 0 \quad x_0 = 1 \rightarrow t = x - 1$$

$$y'' + (t+1)y' + y = 0 \quad , \quad t_0 = 0 \quad y = \sum a_n t^n$$

⑤ شرط في المسألة لحل عادي وفي الآخر تحسب a_1, a_0 .

⑥ كثيرة حدود $\leftarrow$ تساوي معاملات الطرق الأليم.

⑦ دوال مثلثية - لوغاريتمية - أسية $\leftarrow$ تقوم بفكها.

$$* \ddot{y} + \dot{y} - xy = 0 \quad , \quad \underline{y(0)=2}, \quad \underline{y'(0)=1}$$

$\underbrace{\hspace{10em}}_{X_0=0 \text{ ~!~!~!}}$

$$P(x)=1 \rightarrow P(0)=1 \neq \infty \quad \left. \vphantom{P(x)=1} \right\} \rightarrow (0 \cdot P)$$

$$Q(x)=-x \rightarrow Q(0)=0 \neq \infty$$

$$y = \sum a_n x^n$$

$$\dot{y} = \sum n a_n x^{n-1} \rightarrow \ddot{y} = \sum n(n-1) a_n x^{n-2}$$

$$\sum n(n-1) a_n x^{n-2} + \sum n a_n x^{n-1} - \sum a_n x^{n+1} = 0$$

Coeff. of x^0

$$\cancel{2a_2} + 2a_2 + a_1 - \dots = 0 \Rightarrow \boxed{a_2 = \frac{-a_1}{2}}$$

Coeff. of x^1

$$6a_3 + 2a_2 - a_0 = 0 \Rightarrow a_3 = \frac{a_0 - 2a_2}{6}$$

$$\boxed{a_3 = \frac{a_1 + a_0}{6}}$$

→ Coeff. of x^n

$$(n+1)(n+2) a_{n+2} + (n+1) a_{n+1} - a_{n-1} = 0$$

$$a_{n+2} = \frac{-(n+1)a_{n+1} + a_{n-1}}{(n+2)(n+1)}$$

$$\underline{n=2}$$

$$a_4 = \frac{-3a_3 + a_1}{12} = \frac{a_0}{24} + \frac{a_1}{24}$$

$$\underline{n=3} \quad a_5 = \frac{a_0}{120} - \frac{a_1}{120}$$

$$y = a_0 + a_1 x - \frac{a_1}{2} x^2 + \frac{a_1 + a_0}{6} x^3 + \left(\frac{a_0}{24} + \frac{a_1}{24} \right) x^4 + \left(\frac{a_0}{120} - \frac{a_1}{120} \right) x^5 \dots$$

$$y(0) = 2 = a_0$$

حدد بدلالة x

$$\dot{y} = a_1 - a_2 x \dots$$

$$y(0) = a_1 = 1$$

معطى في القانون

$$* (x^2 + 4)y'' + xy' = x + 2, \quad x_0 = 0$$

Sol

$$P(x) = 0 \rightarrow P(0) = 0 \neq \infty$$

$$Q(x) = \frac{x}{x^2 + 4} \rightarrow Q(0) = 0 \neq \infty$$

} $\rightarrow \text{O.P}$

$$y = \sum a_n x^n \rightarrow y'' = \sum n(n-1) a_n x^{n-2}$$

$$\sum n(n-1) a_n x^n + \sum 4n(n-1) a_n x^{n-2} + \sum a_n x^{n+1} = x + 2$$

Coeff. of x^0

$$0 + 8a_2 = 2 \rightarrow a_2 = \frac{1}{4}$$

Coeff. of x^1

$$0 + 24a_3 + a_0 = 1 \Rightarrow a_3 = \frac{-a_0 + 1}{24}$$

Coeff. of x^n

$$n(n-1)a_n + 4(n+2)(n+1)a_{n+2} + a_{n-1} = 0$$

$$a_{n+2} = \frac{-n(n-1)a_n - a_{n-1}}{4(n+2)(n+1)}$$

$$\underline{n=2}$$

$$a_4 = \frac{-2a_2 - a_1}{48} = \frac{\frac{-1}{2} - a_1}{48}$$

$$\underline{n=3} \quad a_5 = \dots$$

$$y = a_0 + a_1 x + \frac{1}{4} x^2 + \frac{-a_0 + 1}{24} x^3 + \frac{\frac{-1}{2} - a_1}{48} x^4 \dots$$

Solve

$$Xy'' + \sin X y = 0$$

$$X_0 = 0$$

$$Xy'' + \left[X - \frac{X^3}{3!} + \frac{X^5}{5!} \dots \right] y = 0$$

$$P(X) = 0 \rightarrow P(0) = 0 \neq \infty$$

$$Q(X) = \left(1 - \frac{X^2}{3!} + \frac{X^4}{5!} \dots \right) \rightarrow Q(0) = 1 \neq \infty$$

$$y = \sum a_n X^n$$

$$\begin{aligned} \therefore \sum n(n-1) a_n X^{n-1} + \sum a_n X^{n+1} - \frac{1}{3!} \sum a_n X^{n+3} \\ + \frac{1}{5!} \sum a_n X^{n+5} \dots = 0 \end{aligned}$$

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Coeff. of x^0

$$0 + \text{لا يوجد حد} + a_{-1} + a_{-3} \dots$$

← لا يوجد معامل لـ x

Coeff. of x^1

$$2a_2 + a_0 = 0 \Rightarrow a_2 = \frac{-a_0}{2}$$

Coeff. of x^n

$$(n+1)na_{n+1} + a_{n-1} - \frac{1}{3!}a_{n-3} + \frac{1}{5!}a_{n-5} \dots = 0$$

$$a_{n+1} = \frac{-a_{n-1} + \frac{1}{3!}a_{n-3} - \frac{1}{5!}a_{n-5} \dots}{n(n+1)}$$

$n=2$

$$a_3 = \frac{-a_1}{6}$$

$n=3$

$$a_4 = \frac{-a_2 - \frac{1}{3!}a_0}{12} = \frac{\frac{a_0}{2} - \frac{a_0}{6}}{12}$$

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$$y = a_0 + a_1 x - \frac{a_0}{2} x^2 - \frac{a_1}{6} x^3 - \dots$$

$$\boxed{2} \text{ S.P} \rightarrow P(0) = \infty \text{ or } Q(0) = \infty$$

$$a) \text{ I.S.P} \rightarrow \left. \begin{aligned} x P(x) \Big|_{x=0} &= \infty \\ \text{or } x^2 Q(x) \Big|_{x=0} &= \infty \end{aligned} \right\} \text{ ليس لها حل}$$

$$b) \text{ R.S.P} \quad x P(x) \Big|_{x=0} \neq \infty$$

$$\text{and } x^2 Q(x) \Big|_{x=0} \neq \infty$$

$$y = \sum a_n x^{n+\lambda}$$

$$y_{G.S} = c_1 y_1 + c_2 y_2$$

$$\begin{array}{ccc} \swarrow & \downarrow & \searrow \\ \lambda_1 - \lambda_2 \neq \text{عدد صحيح} & \lambda_1 = \lambda_2 = \lambda & \lambda_1 - \lambda_2 = \text{عدد صحيح} \end{array}$$

← قوانين هذا ~~part~~ الجزء في المحاضرة

$$* x^2 y'' + x y' + \left(x^2 - \frac{4}{9}\right) y = 0$$

$$P(x) = \frac{1}{x} \rightarrow P(0) = \infty$$

$$Q(x) = 1 - \frac{4}{9x^2} \rightarrow Q(0) = \infty$$

} → S.P

$$\left. \begin{aligned} x p(x) \Big|_{x=0} &= 1 \neq \infty \\ x^2 q(x) \Big|_{x=0} &= x^2 - \frac{4}{9} = -\frac{4}{9} \neq \infty \end{aligned} \right\} \text{R.S.P}$$

$$y = \sum a_n x^{n+2}$$

$$\dot{y} = \sum (n+2) a_n x^{n+2-1}$$

$$\ddot{y} = \sum (n+2)(n+2-1) a_n x^{n+2-2}$$

$$\begin{aligned} &\sum (n+2)(n+2-1) a_n x^{n+2} + \sum (n+2) a_n x^{n+2} \\ &+ \sum a_n x^{n+2+2} - \frac{4}{9} \sum a_n x^{n+2} = 0 \end{aligned}$$

@ $n=0$ smallest $\rightarrow x^2$

$\therefore \text{Coeff } x^2$

$$2(2-1)a_0 + 2a_0 - \frac{4}{9}a_0 = 0$$

$$a_0 \left(2^2 - 2 + 2 - \frac{4}{9} \right) = 0$$

$$a_0 \left(2^2 - \frac{4}{9} \right) = 0$$

$$\lambda^2 = \frac{4}{9} \longrightarrow \lambda_1 = \frac{2}{3} \quad \lambda_2 = -\frac{2}{3}$$

$$\lambda_1 - \lambda_2 = \frac{4}{3} \neq 0$$

→ Coeff. of $x^{\lambda+1}$

$$(\lambda+1) \lambda a_1 + (\lambda+1) a_1 - \frac{4}{9} a_1 = 0$$

$$a_1 \left[\lambda^2 + \lambda + \lambda + 1 - \frac{4}{9} \right] = 0$$

$$\boxed{a_1 = 0}$$

Coeff. of $x^{\lambda+n}$

$$(n+\lambda)(n+\lambda-1) a_n + (n+\lambda) a_n + a_{n-2} - \frac{4}{9} a_n = 0$$

$$a_n = \frac{-a_{n-2}}{(n+\lambda)(n+\lambda-1) + (n+\lambda) - \frac{4}{9}}$$

$$\underline{n=1}$$

$$a_2 = \frac{-a_0}{(2+\frac{2}{3})(2+\frac{2}{3}-1) + (2+\frac{2}{3}) - \frac{4}{9}}$$

$$a_2 = v_0$$

$$y_1 = x^{\frac{2}{3}} \left[a_0 + \frac{-a_0}{\left(2+\frac{2}{3}\right)^2 - \frac{4}{9}} x^2 \dots \right]$$

$$y_2 = x^{-\frac{2}{3}} \left[a_0 + \frac{-a_0}{\left(2-\frac{2}{3}\right)^2 - \frac{4}{9}} x^2 \dots \right]$$

$$y_{G.S.} = C_1 y_1 + C_2 y_2$$

$$\boxed{1-7}$$

$$* x^2 y'' - x y' + (1+x)y = 0$$

$$P(x) = \frac{-1}{x} \rightarrow P(0) = \infty \quad \left. \begin{array}{l} \nexists x P(x)|_{x=0} = -1 \\ \text{s.p.} \end{array} \right\} \begin{array}{l} R \\ \text{e.s.p.} \end{array}$$

$$Q(x) = \frac{1}{x^2} + \frac{1}{x} \rightarrow Q(0) = \infty \quad \nexists x^2 Q(x)|_{x=0} = 1 \neq \infty$$

$$y = \sum a_n x^{n+\lambda}$$

$$y' = \sum (n+\lambda) a_n x^{n+\lambda-1}$$

$$y'' = \sum (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda-2}$$

$$\sum (n+\lambda)(n+\lambda-1) a_n x^{n+\lambda} - \sum (n+\lambda) a_n x^{n+\lambda} + \sum a_n x^{n+\lambda} + \sum a_n x^{n+\lambda+1} = 0$$

$$n=0 \rightarrow x^\lambda \text{ سؤاؤس؟}$$

$$\text{Coeff. of } x^\lambda$$

$$\lambda(\lambda-1)a_0 - \lambda a_0 + a_0 = 0$$

$$a_0 (\lambda^2 - 2\lambda + 1) = 0$$

$$\lambda = \lambda_1 = \lambda_2 = 1$$

* Coeff. of $x^{\lambda+1}$

$$(\lambda+1) \lambda a_1 - (\lambda+1) a_1 + a_1 + a_0 = 0$$

$$\boxed{a_1 = \frac{-a_0}{\lambda^2}}$$

→ Coeff. of $x^{\lambda+n}$

$$(n+\lambda)(n+\lambda-1) a_n - (n+\lambda) a_n + a_n + a_{n-1} = 0$$

$$a_n = \frac{-a_{n-1}}{(n+\lambda)(n+\lambda-1) - (n+\lambda) + 1}$$

$$a_n = \frac{-a_{n+1}}{(n+\lambda)(n+\lambda-2) + 1}$$

$$\underline{n=2} \quad a_2 = \frac{-a_1}{(\lambda+2)(\lambda) + 1} = \frac{-a_1}{(\lambda+1)^2}$$

$$y_1 = x \left[a_0 - a_0 x + \frac{a_0}{4} x^2 \dots \right]$$

$$y(x, \lambda) = x^{\lambda} \left[a_0 - a_0 \lambda^{-2} x - a_0 (\lambda^2 + \lambda)^{-2} x^2 \dots \right]$$

$$x^2 y'' + (x^2 - 2x) y' + 2y = 0$$

$$p(x) = \frac{x^2 - 2x}{x^2} = 1 - \frac{2}{x} \rightarrow p(0) = \infty \rightarrow x p(x) = x - 2 = -2 \neq \infty$$

$$q(x) = \frac{2}{x^2} \rightarrow q(0) = \infty \rightarrow x^2 q(x) = 2 \neq \infty \quad \left. \vphantom{\frac{2}{x^2}} \right\} \text{R-S-P}$$

$$y = \sum a_n x^{n+2}$$

$$y' = \sum (n+2) a_n x^{n+2-1}$$

$$y'' = \sum (n+2)(n+2-1) a_n x^{n+2-2}$$

$$\sum (n+2)(n+2-1) a_n x^{n+2} + \sum (n+2) a_n x^{n+2+1} - 2 \sum (n+2) a_n x^{n+2} + 2 \sum a_n x^{n+2} = 0$$

Coeff. of x^2

$$2(2-1) a_0 + 2a_0 + 2a_0 = 0$$

$$a_0 (2^2 - 2 - 2 \cdot 2 + 2) = 0$$

$$a_0 (2^2 - 3 \cdot 2 + 2) = 0$$

$$a_0 (2-1)(2-2) = 0$$

$$n=1, n=2 \quad - = \text{end} \quad \textcircled{2} \text{ or } 6$$

Coeff. of x^{2+1}

$$2(2+1) a_1 + 2a_0 - 2(2+1) a_1 + 2a_1 = 0$$

$$a_1 = \frac{-2a_0}{2^2 + 2 - 2 \cdot 2 - 2 + 2} = \frac{-a_0}{2-1}$$

Coeff. of X^{n+2}

$$(n+2)(n+2-1)a_n + (n+2-1)a_{n-1} - 2(n+2)a_n + 2a_n = 0$$

$$a_n = \frac{-(n+2-1)a_{n-1}}{(n+2)(n+2-1) + (n+2-1) - 2(n+2) + 2}$$

$$a_n = \frac{-(n+2-1)a_{n-1}}{(n+2-1)(n+2+1) - 2(n+2-1)} = \frac{-a_{n-1}}{n+2-1}$$

$$a_n = \frac{-a_{n-1}}{n+2-1}$$

$$\underline{n=2} \quad a_2 = \frac{-a_1}{2+1} = \boxed{\frac{a_0}{2-1}}$$

$$\underline{\underline{2=2}} \quad y_1 = X^2 \left[a_0 - \frac{a_0}{2-1}X + \frac{a_0}{2-1}X^2 + \dots \right]$$

$$\underline{2=1 \rightarrow a_2 = \infty}$$

$$y(x, 2) = X^2 [a_0 - a_0(2-1)X + a_0(2-1)X^2 + \dots]$$

$$(2-1)y(x, 2) = X^2 [a_0(2-1) - a_0X + a_0(2+1)X^2 + \dots]$$

$$\frac{\partial}{\partial x} = X^2 [a_0 + a_0X^2 + \dots]$$

$$+ X^2 h(x) [a_0(2-1) - a_0X + a_0(2+1)X^2 + \dots]$$

$$y_2 = X [a_0 + a_0X^2 + \dots] + X^2 h(x) [-a_0X + 2a_0X^2 + \dots]$$

$[2a_0 = c_1 g_1 + c_2 g_2]$ *